CDI-II – Prática F.3 16/3/21

$$\frac{d}{dx} f(u(x)) = f'(u(x))\, u'(x)$$

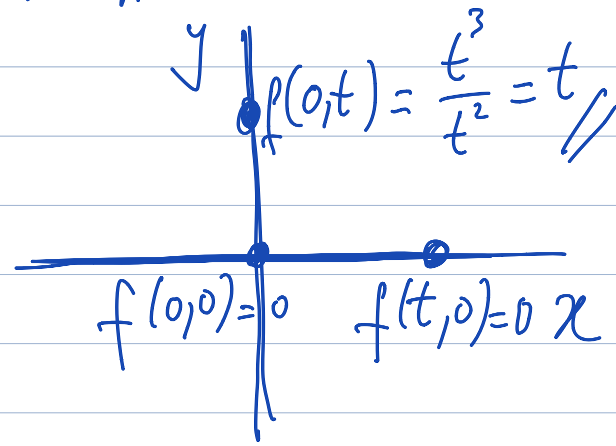
1 – a) $\dfrac{\partial f}{\partial x}(x,y) = \dfrac{2x}{x^2+y^2}$

etc.

2 – $f : \mathbb{R}^2 \to \mathbb{R}$

$\dfrac{\partial f}{\partial x}(0,0)$

$\dfrac{\partial f}{\partial y}(0,0)$



$$\frac{\partial f}{\partial x}(0,0) = \lim_{t \to 0} \frac{f(t,0) - f(0,0)}{t} = 0.$$

$$\frac{\partial f}{\partial y}(0,0) = \lim_{t \to 0} \frac{f(0,t) - f(0,0)}{t} = \lim_{t \to 0} \frac{t}{t} = 1$$

3. Matriz Jacobiana de f em a

$$= Df(a)$$

$$Df(a) = \begin{array}{cc} \begin{bmatrix} & \vdots & \\ \cdots\cdots & \dfrac{\partial f_j}{\partial x_k}(a) & \cdots\cdots \\ & \vdots & \end{bmatrix} & \leftarrow j \\ \uparrow k & \end{array}$$

3-a) $Df(x,y) = \begin{bmatrix} y & x \\ \frac{y}{xy} & \frac{x}{xy} \end{bmatrix}_{2\times 2}$

$= \begin{bmatrix} y & x \\ \frac{1}{x} & \frac{1}{y} \end{bmatrix}.$

etc.

. . .

3-e) $D\gamma(t) \equiv \gamma'(t) = \begin{bmatrix} 3t^2 \\ -e^{-t} \\ -\frac{1}{t^2} \end{bmatrix}_{3\times 1}$

4 – Se $f: \mathbb{R}^n \to \mathbb{R}^m$ for diferenciável em a, então dado $v \in \mathbb{R}^n$

$$\underbrace{\frac{\partial f}{\partial v}(a)}_{\text{pedida}} = \underbrace{Df(a)\, v}_{\text{a calcular}}$$

4-a) y^x, $y > 0$

$y^x = e^{x \ln y}$

$$\boxed{f(x,y) = e^{x \ln y}}$$

etc.

5 - $f(x,y) = xy^2 + x^2y \qquad (1,2)$

$0 = \frac{\partial f}{\partial v}(1,2) = Df(1,2)v \qquad v\,?$

$$\boxed{0 = Df(1,2)\,v} \quad ?$$

Calcular e determinar v.

$$Df(1,2) = \begin{bmatrix} \frac{\partial f}{\partial x}(1,2) & \frac{\partial f}{\partial y}(1,2) \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \end{bmatrix}.$$

$v = (x,y) \qquad 8x + 5y = 0$

ou $(8,5)\cdot(x,y) = 0$

$x = -5, \quad y = 8$

$v = (-5, 8)$ ✓

——— || ———

6 – $f(x,y) = y\,\|(x,y)\|$

∇f não é dif. na origem.

$\Rightarrow$ temos de usar a definição de f. diferenciável.

$$f(a+h)-f(a)-\underbrace{Df(a)}_{\text{matriz das derivadas parciais.}}h = o(h)$$

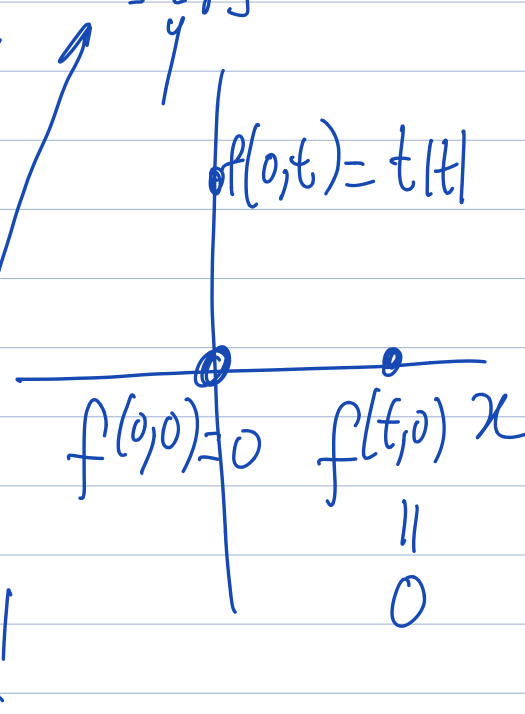
$a=(0,0) \qquad h=(x,y)$

$$f(x,y)\ \cancel{-\underbrace{f(0,0)}_{=0}-\underbrace{Df(0,0)}_{\begin{bmatrix}0 & 0\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=0}(x,y)} = o(x,y)$$

$$\frac{\partial f}{\partial x}(0,0) = 0$$

$$\frac{\partial f}{\partial y}(0,0) = \lim_{t\to 0}\frac{t|t|}{t}$$

$$= \lim_{t\to 0}|t| = 0\ !$$



temos de verificar que

$$f(x,y) = o(x,y)$$

$$\lim_{(x,y)\to(0,0)} \frac{|f(x,y)|}{\|(x,y)\|} \overset{?}{=} 0$$

$$\lim_{(x,y)\to(0,0)} \frac{|y|\,\cancel{\|(x,y)\|}}{\cancel{\|(x,y)\|}} = 0. \checkmark$$

$\therefore f$ é dif. em $(0,0)$ e

$$Df(0,0) = \begin{bmatrix} 0 & 0 \end{bmatrix}.$$

//

Se $(x,y) \neq (0,0)$, f é dif. em (x,y)

e $Df(x,y) = \left[\frac{\partial f}{\partial x}(x,y) \quad \frac{\partial f}{\partial y}(x,y) \right]$

$$\frac{\partial f}{\partial x}(x,y) = \frac{\partial}{\partial x}\left(y\sqrt{x^2+y^2} \right)$$

$$= \frac{\partial}{\partial x}\left(y\,(x^2+y^2)^{1/2} \right)$$

y constante !

$$= y\,\frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}\,2x$$

$$= \frac{xy}{\sqrt{x^2+y^2}}$$

etc.

$$\frac{\partial f}{\partial y}(x,y) = \frac{\partial}{\partial y}\left(y\,(x^2+y^2)^{1/2}\right) \qquad x \text{ constante}$$

$$= \sqrt{x^2+y^2} + y\,\frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}\,2y$$

$$= \sqrt{x^2+y^2} + \frac{y^2}{\sqrt{x^2+y^2}}\,.$$

etc.

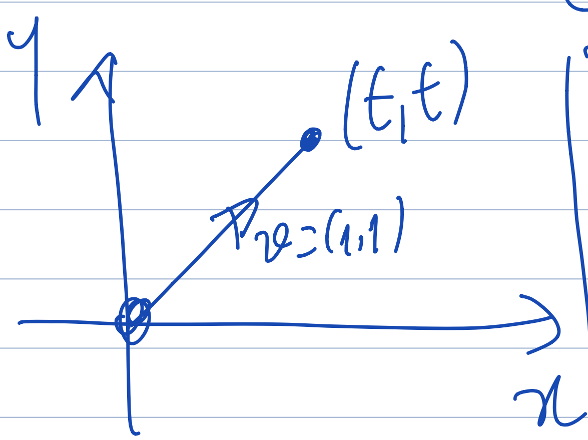
7- $f(a+h) - f(a) - Df(a)h = o(h)$

7-i) f não é contínua em $(0,0)$, logo não é dif. em $(0,0)$

7-ii) Calcular $\frac{\partial g}{\partial v}(0,0)$, $v = (1,1)$

e verificar que $\frac{\partial g}{\partial v}(0,0) \neq 0$.

$$\begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 0$$



$$\lim_{t \to 0} \frac{g(t,t) - g(0,0)}{t} =$$

$$\lim_{t \to 0} \frac{\frac{t^2}{\sqrt{2}|t|}}{t}$$ não existe

7 - iii) $\frac{\partial h}{\partial x}(0,0) = 0$; $\frac{\partial h}{\partial y}(0,0) = 0$

$$\underbrace{h(x,y) - \cancel{h(0,0)}}_{} - \cancel{Dh(0,0)(x,y)} = o(x,y)$$

$$\cancel{h(0,0)} = 0 \qquad \cancel{Dh(0,0)(x,y)} = 0$$

$$\frac{|h(x,y)|}{\|(x,y)\|} = \frac{\frac{x^2y^2}{x^4+y^2}}{\|(x,y)\|} = \frac{|x|\,|x|\,y^2}{\|(x,y)\|\,(x^4+y^2)}$$

$$\frac{|x|}{\|(x,y)\|} \leq 1 \qquad \frac{y^2}{x^4+y^2} \leq 1$$

$$\leq |x| \longrightarrow 0$$

$$y^2 \leq y^2 + x^4$$

8-a) $(0,1) \neq (0,0)$

$\rightarrow$ seguir as regras de derivação.

8-b) $(0,1) \neq (0,0)$

$v = (2,1)$

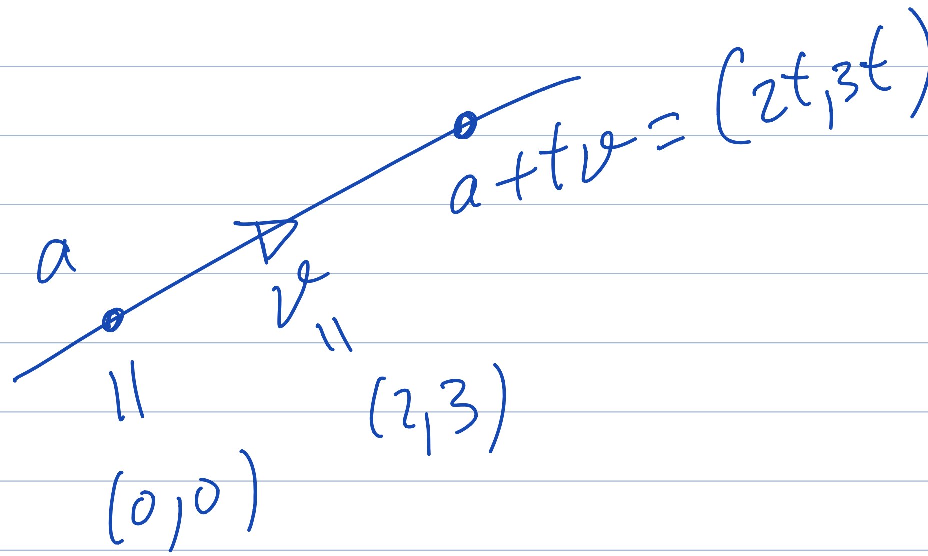
$a = (0,1)$

$$\frac{\partial f}{\partial v}(0,1) = \begin{bmatrix} \frac{\partial f}{\partial x}(0,1) & \frac{\partial f}{\partial y}(0,1) \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

✓

8-c) usar a definição:

$$\frac{\partial f}{\partial v}(a) = \lim_{t \to 0} \frac{f(a+tv) - f(a)}{t}$$



$$\lim_{t \to 0} \frac{f(2t,3t)}{t} \qquad \frac{18}{13} \neq 0$$

$$= \lim_{t \to 0} \frac{\frac{2t(9t^2)}{4t^2+9t^2}}{t} = \lim_{t \to 0} \frac{18t^3}{13t^3}$$

por outro lado

$$\frac{\partial f}{\partial x}(0,0) = 0, \quad \frac{\partial f}{\partial y}(0,0) = 0$$

$$Df(0,0)\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = 0$$

$$\frac{\partial f}{\partial v}(0,0) = \frac{18}{13} \neq 0$$

$\Rightarrow$ f não é dif. em (0,0).